Home Health Care Services: Models and Algorithms

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Outline

- **Motivation and three main problems**
- Models and algorithms
- Experiments with real-world data
- Summary and outlook

Motivation

- Aging society
- Decreasing budget per patient
- Shift from hospital care towards home care
- Intensive competition among home health care (HHC) service providers
- People in rural areas live in self-owned houses (which have a small value)

→ **Generating improved schedules for HHC services**

- Improvement of utilization of resources to reduce costs
- Higher level of comfort for the patients

Motivation (2)

HHC service “We care 4 you”

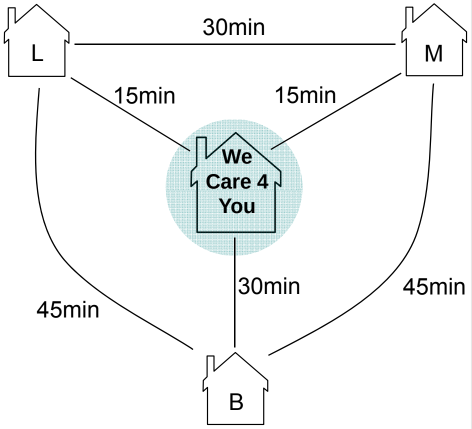
- Patients

Patient	Service duration	Time window
Bart	30 min	8:00 – 8:45
Liza	30 min	8:30 – 9:30
Maggie	30 min	8:00 – 9:30

- Distances

	Bart	Liza	Maggie
Bart	0	45	45
Liza	45	0	30
Maggie	45	30	0

- Nurses: Patty, Selma



Motivation (3)

HHC service “We care 4 you”

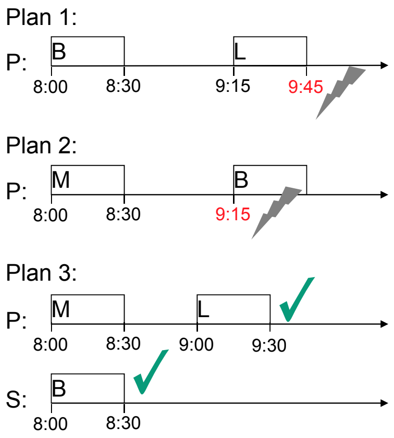
- Patients

Patient	**Service duration**	**Time window**
Bart	30 min	8:00 – 8:45
Liza	30 min	8:30 – 9:30
Maggie	30 min	8:00 – 9:30

- Distances

	Bart	**Liza**	**Maggie**
Bart	0	45	45
Liza	45	0	30
Maggie	45	30	0

- Nurses: Patty, Selma



Motivation (4)

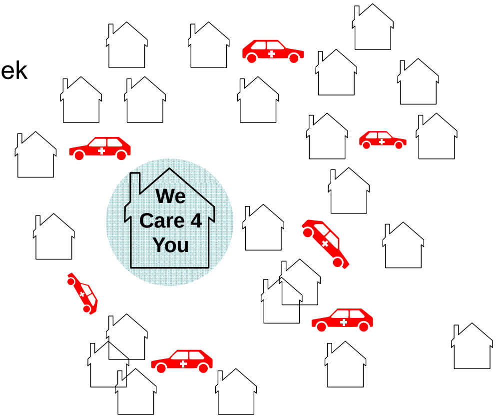
Exemplary HHC service

Patients: 50 = 300 visits per week

- Time windows
- Different shift combinations
- Qualifications

Nurses: 15

- Different availabilities
- Different qualifications



Three main problems

- Home health care problem (HHCP)
  → Feasible assignment of jobs to nurses and start times to jobs
- Master schedule problem (MSP)
  → Mid-term plan as basis for weekly assignment of nurses to jobs
- Operational planning problem (OPP)
  → Incorporate upcoming events (perturbations) into an existing schedule

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Outline

Models and algorithms – HHCP

Home Health Care Problem (HHCP)

- Combination of two $\mathcal{NP}$-hard problems
  - Periodic Vehicle routing problem with time windows
  - Nurse rostering problem
- Task: Feasible assignment of jobs to nurses and start times to jobs
- Given
  - Planning horizon of $\mathcal{S} = \{1, \ldots, S\}$ shifts with hard time windows $[0, H]$
  - Set of nurses $\mathcal{N} = \{1, \ldots, N\}$
  - Set of jobs $\mathcal{J} = \{1, \ldots, J\}$
  - Travel distance between jobs $D = (d_{i,k})_{i,k \in \mathcal{J}}$

Models and algorithms – HHCP

- Objective function: Minimize the weighted sum of…
  - … the number of unscheduled tasks
  - … patient-nurse loyalty penalty
  - … overtime costs
  - … travelling distance

$$\begin{aligned} \text{min} \quad & \alpha_1 \cdot DNC \\ & + \alpha_2 \cdot \sum_{j=1}^{J} \Big(|X_j| - 1 \Big) \\ & + \alpha_3 \cdot \sum_{n=1}^{N} \max \Big\{ 0, \sum_{s \in \mathcal{S}} \Big(end_n^s - start_n^s \Big) - wt_n \Big\} \cdot c_n \\ & + \alpha_4 \cdot \sum_{n=1}^{N} \sum_{s=1}^{S} dt_n^s \end{aligned}$$

- Exemplary overview of constraints
  - Exactly one of the possible shift combinations is chosen, and a task representing the job is scheduled in each of those shifts.
  - Each job is performed during its time window.
  - A nurse can only be assigned to a job if she possesses the required qualification.
  - Each route starts and ends at the depot within the shift‘s time window

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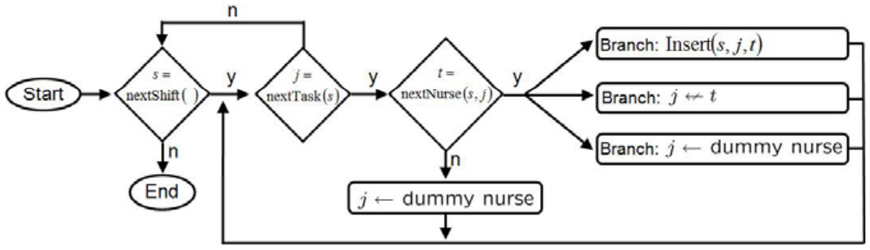
Models and algorithms – HHCP

- Solution approach: Two-stage algorithm

  I. Constraint programming heuristic



→ Quickly generate a feasible solution

Models and algorithms – HHCP

II. Adaptive large neighborhood search (ALNS)

```
function ALNS(solution $s, q \in \mathbb{N}, \sigma \in \mathbb{N}$)
    Solution $s_{best} = s$
    $m = 0$
    repeat
        $m = m + 1$
        $s' = s$
        Choose removal operation $r$ with probability $\pi_r$
        Chose insertion operation $i$ with probability $p_i$
        Remove $q$ requests from $s$‘ with operation $r$
        Reinsert removed requests into $s$‘ with operation $i$
        if $f(s') < f(s_{best})$ then
            $s_{best} = s'$
        if accept$(s', s)$ then
            $s = s'$
        if $(m \bmod \sigma) == 0$ then
            Update probabilities for operations
    until (stopping-criterion is met)
    return $s_{best}$
```

→ CP layer maintains feasibility while improving the solution

Models and algorithms – HHCP

- Removal heuristics
  - Random removal
  - Shift combination removal
  - Worst removal
- Insertion heuristics
  - In order insertion
  - Greedy insertion
- Score for each insertion and removal heuristic that stores how successful the heuristic was in the past

→ Preference of previously successful operations

Models and algorithms – MSP

Master schedule problem (MSP)

- Task: Mid-term plan as basis for weekly assignment of nurses to jobs
- Given
  - Planning horizon of $\mathcal{S} = \{1, \ldots, S\}$ shifts with hard time windows $[0, H]$
  - Set of jobs $\mathcal{J} = \{1, \ldots, J\}$
  - Travel distance between jobs $D = (d_{i,k})_{i,k \in \mathcal{J}}$
  - Maximal tour length L
- Objective function: Minimize the number of tours required to visit all patients

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Models and algorithms – MSP

- Exemplary overview of constraints
  - Each job must be performed as often as indicated by its frequency
  - Each task is assigned to exactly one tour in each shift
  - Each tour has a maximal length
  - A tour only contains tasks of one L qualification dimension
  - Each tour starts and ends at the depot within the shift's time window
- Jobs only have one possible shift combination
- Solution approach:
  - Constraint programming heuristic
  - Generate lower and upper bounds on the minimal number of tours

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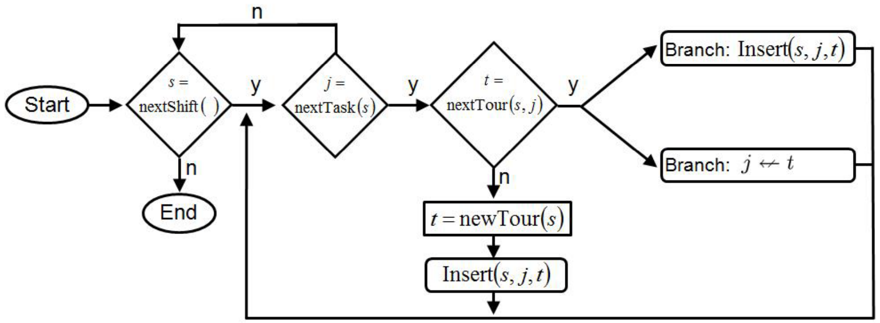
Models and algorithms – MSP

- Constraint programming heuristic



→ Construct tours such that next task is scheduled at closest tour that is compatible with required qualification

Models and algorithms – OPP

Operational planning problem (OPP)

- Task: Incorporate upcoming events (perturbations) into existing schedule
  - Non-availability of personnel
  - Changes in jobs
  - **Incorporation of a new patient**
- Indicators to measure perturbation: Inconvenience for…
  - … nurses
  - **… patients**

$$\varphi_j^s = \begin{cases} 0 & \text{, if } x_j^s = 0, \\ \left|st_j^s - \widehat{st}_j^s\right| & \text{, if } x_j^s \neq 0, \hat{x}_j^s \neq 0, \\ 0 & \text{, if } x_j^s \neq 0, \hat{x}_j^s = 0 \text{ and } st_j^s \in [hbs_j, hbe_j - pt_j] \\ \min\left\{\left|st_j^s - hbs_j\right|, \right. & \\ \qquad \left. \left|st_j^s + pt_j - hbe_j\right|\right\} & \text{, if } x_j^s \neq 0, \hat{x}_j^s = 0 \text{ and } st_j^s \notin [hbs_j, hbe_j - pt_j]. \end{cases}$$

→ **Deviation of start times of initial to new solution**

Models and algorithms – OPP

- Objective function: Minimize…
  - … perturbation penalty φ_j^s
  - … the original objectives of the HHCP $\widehat{\zeta}$

$$\min \; \beta_1 \cdot \widehat{\zeta} + \beta_2 \cdot \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{J}} \varphi_j^s$$

- Solution approach: Two-stage algorithm

I. Constraint programming (CP) insertion heuristic

  - Time window of the new task ends before the nurse-shift starts
  - Nurse-shift ends before the time window of the new task starts
  - Time window of the new task (partly) overlaps the nurse-shift

→ **Quickly generate a feasible solution**

Models and algorithms – OPP

II. Tabu search algorithm

```
Input: OPP $\Theta$, Initial solution $\sigma$, Move-List $\mathcal{M} = \{1, \dots, M\}$
$k = 0, T = \emptyset, \sigma^* = \sigma$
while termination criterion not satisfied do
    $k = k + 1$
    Select $m \in \mathcal{M}$
    if $N_m(\sigma) \setminus T \neq \emptyset$ then
        $\sigma_k = \operatorname{argmin}_{s \in N_m(\sigma)} \{\zeta(s)\}$
        if $\zeta(\sigma_k) < \zeta(\sigma^*)$ then
            $\sigma^* = \sigma_k$
        end if
        $\sigma = \sigma_k$
        Update $T$ mit $\sigma_k$
    end if
end while
```

→ **Improve the solution until a termination criterion (time limit or move limit) is met**

→ **CP layer maintains feasibility while improving the solution**

Outline

- Motivation and three main problems
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- **Experiments on real-world data**
- Summary and outlook

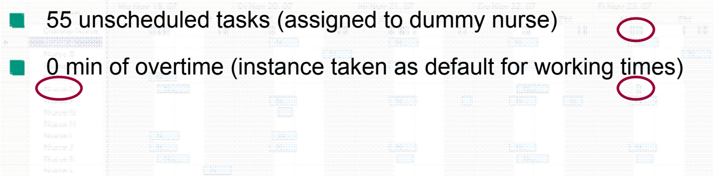
Experiments on real-world data

- Two test instances
  - I. Health care service provider in a rural area in Germany
    - Long traveling distances up to 45 minutes
    - Many short but also several long service times (> 3 hours)
  - II. Health care service provider in an urban area in the Netherlands
    - Short traveling distances
    - Only short service times (≤ 45 minutes)

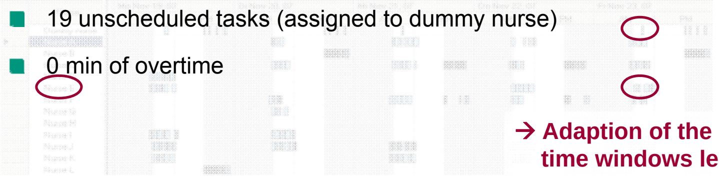
Instance	Nurses	Patients	Tasks
D	11	53	287
NL	12	95	361

Experiments on real-world data

- Solution created by commercial software
  - 55 unscheduled tasks (assigned to dummy nurse)
  - 0 min of overtime (instance taken as default for working times)



- Solution created by the ALNS for the HCCP
  - 19 unscheduled tasks (assigned to dummy nurse)
  - 0 min of overtime

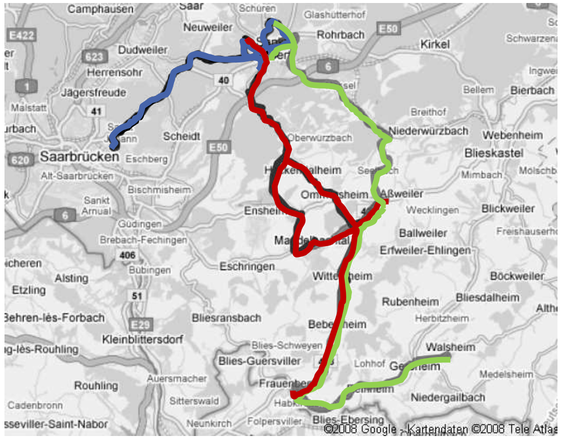


→ Adaption of the nurses' time windows leads to an improved schedule

- Termination criterion: 1000 moves
- Initial solution: 11 sec

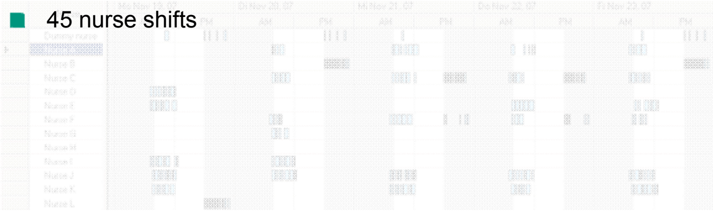
Experiments on real-world data

- Example of three routes created by the ALNS



Experiments on real-world data

- Solution created by the ALNS for the HHCP
  - 45 nurse shifts



- Solution created by the CP heuristic for the MSP
  - 53 tours

→ Negotiation of time windows and nurse shifts can reduce the number of tours

- Computation time: 19.5 sec

Outline

- Motivation and three main problems
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- Experiments on real-world data
- **Summary and outlook**

Summary

- Set of algorithms to support the planning process of HHC services
  - Complete re-creation of a schedule
  - Creation of a master schedule and incorporation of unforeseen events
- Introduction of shift combinations for jobs
- Definition of patient-nurse loyalty: Minimize the number of different nurses per patient throughout the schedule
- Development of hybrid CP-meta-heuristic algorithms
- Development of new operators and a new application for ALNS
- Proved applicability to real-world data sets

Outlook: Further research topics

- Home health care problem
  - Introduction of split services
  - Implementation of further propagation techniques (whenDomain)
- Master schedule problem
  - Minimization of qualification level for a tour
  - Introduction of a second target length for a route
- Operational planning problem
  - Development of a decision support system
  - Incorporation of further events into the OPP

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Thank you for your attention!

http://www.itwm.fhg.de/zentral/download/berichte/bericht_173.pdf

Backup

Literature Review

Home health care problem (HHCP)

- E. Cheng and J. L. Rich. A Home Health Care Routing and Scheduling Problem. Technical Report TR98-04, Department of CAAM, Rice University, USA, 1998.
- S. Bertels and T. Fahle. A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem. *Computers & Operations Research*, 33:2866-2890, 2006.
- P. Eveborn, P. Flisberg, and M. Rönnqvist. LAPS-CARE – an operational system for staff planning of home care. *European Journal of Operational Research*, 171:962-976, 2006.

Literature Review

Master schedule problem (MSP)

- J. Desrosiers, M. Sauvé, and F. Soumis. Lagrangian relaxation methods for solving the minimum fleet size multiple travelling salesman problem with time windows. *Management Science*, 34(8):1005-1022, August 1988.
- M. Surico, U. Kaymak, D. Naso, and R. Dekker. Hybrid Meta-Heuristics for Robust Scheduling. Technical report, Erasmus University Rotterdam, Netherlands, 2006.
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Operational planning problem (OPP)

- R. Barták, T. Müller, and H. Rudová. Minimal Perturbation Problem – A Formal View. *Neural Network World*, 13(5):501-511, 2003.
- H. E. Sakkout, T. Richards, and M. G. Wallace. Minimal Perturbation in Dynamic Scheduling. In *13th European Conference on Artificial Intelligence, Brighton, UK*, pages 504-508. John Wiley and Sons, 1998.
- J.-F. Cordeau, G. Laporte, and A. Mercier. A unified tabu search heuristic for vehicle routing problems with time windows. *Journal of Operational Research Society*, 52:928-936, 2001.

Models and algorithms – HHCP

Example

- Constraint: Exactly one of the possible shift combinations is chosen

$\forall$ Jobs j :

$$element(\rho_j, r_j, R_{j,1}, \dots, R_{j,K_j}) =$$
$$\left\{(e, f, d_1, \dots, d_n) : e \in D(\rho_j), f \in D(r_j), \ \forall i : d_i \in D(R_{j,i}), f = d_e\right\}$$

- Propagation:

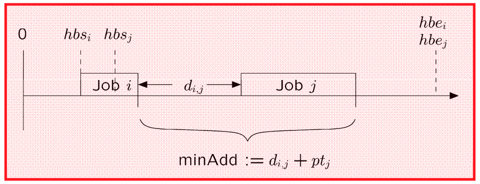
$$\forall s \in r_j: \quad z_j^s = 1, \quad D(x_j^s) = D(x_j^s) \setminus \{-1\}.$$

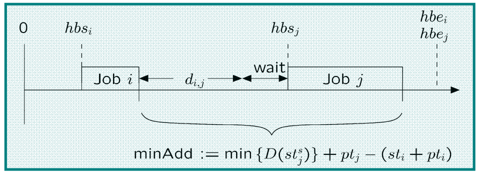
$$\forall t \notin r_j: \quad z_j^t = 0, \quad x_j^t = -1, \quad st_j^t = \min\left\{D(st_j^t)\right\}$$

Models and algorithms – MSP

- Algorithm getNextTour(Shift s, Job j)

```
$D = \infty$, $T$ = -1
for all $t \in \mathcal{T}$
    if $y_t \neq s$ then next $t$
    if $tq_t \parallel req_j$ then next $t$
    if $t \notin D(x_s^j)$ then next $t$
    $i$ = currently last job on tour $t$
    $\text{minAdd} := \max\{\boxed{d_{i,j} + pt_j}, \boxed{\min\{D(st_j^s)\} + pt_j - (st_i^s + pt_i)}\}$
    if $dur_t$ + minAdd $\leq L$ then
        if $d_{i,j} < D$ then
            $D = d_{i,j}$, $T = t$
        else if $d_{i,j} = D$ then
            if $dur_t > dur_T$ then
                $D = d_{i,j}$, $T = t$
return $T$
```





Computation

- AMD Athlon X2
- 2.1 GHz
- 2 GB RAM
- Screenshots: ILOG Scheduler Viewer 1.0